Logic Worksheet

INTRODUCTION

Validity:
A deductive argument is **valid** if it has a form that would make it impossible for the premises to be true and the conclusion false. If a deductive argument is valid, then its premises' being true would guarantee that its conclusion is true.

To test whether or not an argument is valid, you should first imagine that the premises are true—whether or not they actually are—and then ask yourself, without appealing to any other knowledge you have, could you still imagine the conclusion being false? If you can, the argument is invalid. If you can't, then the argument is valid.

Note that validity is a matter of the form or structure of an argument, as opposed to the content. If an argument is valid, then any other argument with the same logical structure will also be valid, regardless of its content. Also, keep in mind that an argument can be valid even if its premises are not actually true.

An argument that has true premises (regardless of whether it is valid or invalid) is said to be **factually correct**.

An argument that is *both* valid and factually correct is **sound** .

Some hints on determining validity:
When you are checking the validity of an argument, you may need to visualize what the world would have to be like if its premises were true. Sometimes Venn diagrams can prove helpful. Consider this argument, for instance:
1. All dogs are cats
2. All cats are lizards
3. Therefore, all dogs are lizards

Clearly, this argument is not factually correct, for the premises are false. But it may be of a valid argument form. To check this, we must imagine a possible world in which all the premises are true. So consider premise 1. We can represent what it is saying by drawing two circles. One circle represents a collection of all the dogs in the possible world, and the other circle represents all the cats. Since the premise says that ALL dogs are cats, we know that every member in the circle of cats must also be a member in the circle of dogs. So we must put the dog circle INSIDE the cat circle. Keep in mind that there are no premises telling you that all CATS are DOGS. Thus, there should be some leftover area of the cat circle that falls outside of the dog circle, to show that there may be some cats that are not dogs.

Now, look at the second premise. If all cats are lizards, then the whole CAT circle (with the DOG circle still inside it) must be placed within the circle of all the lizards in the world. At this point, we should have an accurate representation of the premises. Do they guarantee the conclusion? That is to ask: is it possible in that world for the conclusion to be false? Since you will notice that there is no area of the dog circle outside the lizard circle, you should see that if these premises were true, the conclusion must also be true. The argument is therefore valid.

"if…then" premises:
It may be possible to use Venn diagrams to help clarify "if…then" premises. Suppose we have a premise that says "if P then Q". We can think of this as stating that whenever P is true, then Q must also be true. Another way of putting it is by saying: All cases of P are also cases of Q. If you find the Venn diagrams helpful, you could represent this by drawing a large Q-circle with a smaller P-circle inside of it. Notice that this leaves with some area of the Q-circle that is not also in the P-area. That is because "if P then Q" does not mean that there cannot be instances where Q is true, but P is false. An important thing to notice, however, is that if you say that Q is false, then you must also say that P is false. Perhaps you could represent a statement (such as 'P' or 'Q') as being false by crossing out the area of its circle. So, for example, if another premise says that Q is false (or simply not-Q) then you could draw an X through the whole Q circle. Of course, this means also drawing an X through the P circle as well, so P must be false too.

Below are some arguments. For each argument try to determine whether or not it is valid (you may want to take note of whether or not you think the argument is sound as well). It is worth taking the time to symbolize each argument (for instance, using 'P's and 'Q's to stand for statements. Pay attention to which symbolized arguments are valid and which are invalid. Doing so will help you recognize valid and invalid arguments with greater ease. I have included answers with some comments following the exercises.

EXERCISES:

A.
1. If Jane has a cat, then Jane has a pet
2. Jane has a cat
3. Therefore, Jane has a pet

B.
1. If Jane has a cat, then Jane has a pet
2. Jane has a pet
3. Therefore, Jane has a cat

C.
1. If Jane has a cat, then Jane has a pet
2. It is not the case that Jane has a pet
3. Therefore, it is not the case that Jane has a cat

D.
1. If Jane has a cat, then Jane has a pet
2. It is not the case that Jane has a cat
3. Therefore, it is not the case that Jane has a pet

E.
1. If pigs fly, then hell has frozen over
2. Pigs fly
3. Therefore, hell has frozen over

F.
1. If Bush is president, then a Republican is president
2. A Republican is president
3. Therefore, Bush is president

G.
1. If E.T. phones home, then blue is Joe's favorite color
2. It is not the case that blue is Joe's favorite color
3. Therefore, it is not the case that E.T phones home

H.
1. It is not the case that Yoda is green
2. If Darth Vader is Luke's Dad, then Yoda is green
3. Therefore, it is not the case that Darth Vader is Luke's dad

I.
1. Dan plays the cello
2. If Mary plays the harp, then Owen plays the clarinet
3. Therefore, it is not the case that Mary plays the harp

J.
1. All smurfs are snorks
2. All ewoks are snorks

3. Therefore, All smurfs are ewoks

K.
1. Kate is a lawyer
2. Therefore, Kate is a lawyer

L.
1. If it is morally permissible to kill an 8-month old fetus, then it is morally permissible to kill a newborn infant
2. It is not the case that it is morally permissible to kill a newborn infant
3. Therefore, it is not the case that it is morally permissible to kill an 8-month old fetus

M.
1. If Rufus is a human being, then Rufus has a right to life
2. It is not the case that Rufus is a human being
3. Therefore, it is not the case that Rufus has a right to life

N.
1. All anarchists are socialists
2. All socialists are totalitarians
3. Therefore, all anarchists are totalitarians

O.
1. No cat is a biped
2. All kangaroos are bipeds
3. Therefore, No cat is a kangaroo

P.
1. If there is order in the universe, then God exists
2. There is order in the universe
3. Therefore, God exists

Q.
1. Amy joins the Army, or Mary joins the Marines
2. It is not the case that Mary joins the Marines
3. Therefore, Amy joins the Army

(Note: the word 'OR' is a logical term much like 'if…then', 'therefore' and 'it is not the case that…' Like these other terms, 'OR' is part of the structure or form of the argument, rather than the content.)

R.
1. Ariel joins the Air Force or Nancy joins the Navy
2. Nancy joins the Navy
3. Therefore, Ariel joins the Air Force

ANSWERS:

A.
1. If P then Q
2. P
3. Therefore, Q
Valid (Modus Ponens)

B.
1. If P then Q
2. Q
3. Therefore, P
Invalid
This argument form is commonly mistaken as being valid. Notice that even if the premises are true, the conclusion could still be false: Jane could have a dog.

C.
1. If P then Q
2. Not: Q
3. Therefore, Not: P
Valid (Modus Tollens)

D.
1. If P then Q
2. Not: P
3. Therefore, Not: Q
Invalid
This is another argument form that is commonly mistaken as being valid. Again, Jane could still have a pet even if she does not have a cat, maybe she has a bird. Her owning a bird is not ruled out by the premises.

E.
1. If P then Q
2. P
3. Therefore, Q
Valid (Modus Ponens)
Notice that this argument is still valid even though (as far as we know) all the premises (and the conclusion) are, in fact, false.

F.
1. If P then Q
2. Q
3. Therefore, P
Invalid
This is the same invalid form as argument B. Notice that all the premises and the conclusion are in fact true. Still, the argument is invalid: it is possible for all the premises to be true and the conclusion still be false. You can imagine a world in which the two premises are true, and yet George Bush is not president. Some other Republican could be president.

G.
1. If P then Q
2. Not: Q
3. Therefore, Not: P
Valid (Modus Tollens)
This is the same argument form as argument C. This seem trickier than argument C since premise (1) in argument G asserts an unlikely relationship between what Joe's favorite color is and whether or not E.T. phones home. What could those two things have to do with one another? They probably have nothing to do with one

another. Therefore, premise (1) is probably false. But to check the argument for validity we need to imagine that it is true. So we need to imagine that somehow, for some reason unbeknownst to us, if it is true that E.T. phones home, then it also will be true that Joe's favorite color is blue.

H.
1. Not: P
2. If Q then P
3. Therefore, Not: Q
Valid (Modus Tollens)
This is the same argument form as argument C and G. The only difference is that the if-then statement is the second premise rather than the first. That's okay, the order of the premises is unimportant for determining validity. Also, don't be fooled by the actual falsity of the premises: IF they were true, the conclusion would have to be true as well.

I.
1. P
2. If Q then R
3. Therefore, Not: Q
Invalid

J.
1. All x are y
2. All z are y
3. Therefore, x are z
Invalid. You can see this by considering an argument of the same logical form that has premises that are easier to imagine being true (because they are true): 1. All humans are primates. 2. All gorillas are primates. 3. Therefore, all humans are gorillas.

K.
1. P
2. Therefore, P
Valid
Obviously, if "Kate is a lawyer" is true, then it would be impossible for "Kate is a lawyer" to also not be true. But is this because of the logical form of the argument? Well, try uniformly substituting different sentences for 'P' and see what happens. (Remember, whatever you substitute for 'P' must go everywhere there is a 'P'.) However, this argument does *beg the question*, but that's a different question from the question of validity and invalidity.

L.
1. If P then Q
2. Not: Q
3. Therefore, Not: P
Valid (Modus Tollens)
Same argument form as C, G, and H.

M.
1. If P then Q
2. Not: P
3. Therefore, Not: Q
Invalid
Same invalid argument form as in argument D. Even if the premises are true, it is still possible that other life-forms besides human beings have a right to life. It is quite plausible to suppose at the very least that chimpanzees have a right to life.

N.
1. All x are y
2. All y are z

3. Therefore all x are z
Valid

O.
1. No x is y
2. All z are y
3. Therefore, no x is z
Valid. If it is hard to see why, try drawing a Venn diagram.

P.
1. If P then Q
2. P
3. Therefore, Q
Valid (Modus Ponens)

Q.
1. P or Q
2. Not: Q
3. Therefore, P
Valid.

R.
1. P or Q
2. Q
3. Therefore, P
Invalid. The premises don't guarantee that Ariel joined the Air force (though he might have.) Note: In logic, the word 'or' is usually understood in its INCLUSIVE sense. You should understand the first premise as saying something to the effect of: "either Ariel joins the air force or Nancy joins the Navy or both".
